HEATING OF A SEMIINFINITE BODY BY A FINITE HEAT SOURCE IN THE SHAPE OF A SQUARE

V. P. Kozlov, V. N. Lipovtsev,
and G. P. Pisarik

UDC 536.2.083

We determine the three-dimensional, unsteady temperature field in a semiinfinite body heated through a square region on its surface. The heat flux through the square is taken to be an arbitrary function of time.

The time and space dependence of temperature fields in semiinfinite bodies heated by finite heat sources of various geometrical shapes (circle, ring, strip, etc.) forms the theoretical foundation for certain nondestructive methods (methods that do not destroy the integrity of the sample) of testing the thermal and physical characteristics of materials [1-3]. In the present paper we consider the three-dimensional, unsteady heat equation for a semiinfinite (in the thermal sense) body heated through a finite square region on its surface with a heat flux density $q(\tau)$ which is an arbitrary function of time.

Formulation of the Problem. We consider a semiinfinite body with a uniform initial temperature T_0 = const. For times $\tau > 0$ part of the surface of the body bounded by a square of side $2l$ is heated by a specific heat flux density $q(\tau)$ which is an arbitrary function of time. The rest of the surface is thermally insulated. The problem is solved using Cartesian coordinates, where the origin of the coordinate system ($x = y = z = 0$) is chosen on the surface of the body in the center of the heated square region (see Fig. 1). It is required to find the temperature field on the axis $x = y = 0$, $z \geqslant 0$ of the body, and in particular at the center ($x = y = z = 0$) of the square on the surface of the body.

If we formulate the problem in terms of the temperature differences $\Theta_i(x, y, z, \tau) = T_i(x, y, z, \tau) - T_0$ with zero initial conditions for the Θ_i, then it will be necessary to find the solution of a system of four heat equations

$$a\nabla^2\Theta_i(x, y, z, \tau) = \frac{\partial\Theta_i(x, y, z, \tau)}{\partial\tau}, \tag{1}$$

where $i = 1, 2, 3, 4$; $\nabla^2\Theta_i(x, y, z, \tau)$ is the Laplacian in Cartesian coordinates. The problem is subject to the initial conditions

$$\Theta_i(x, y, z, \tau) = 0 \tag{2}$$

and the boundary conditions

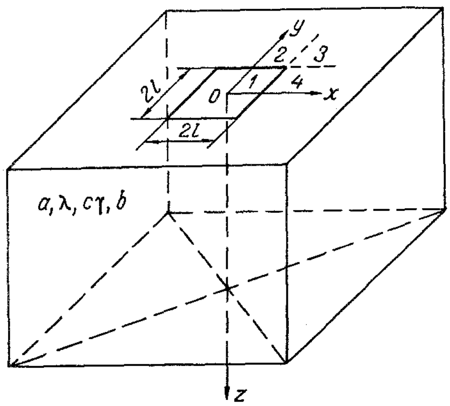

Fig. 1. Model of a semiinfinite body (in the thermal sense) heated through a square region on its surface by a heat flux density $q(\tau)$ which is an arbitrary function of time. The rest of the surface: $z = 0$, $|x| > l$, $|y| > l$; is assumed to be thermally insulated.

Economics Institute, State Planning Committee, BSSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 6, pp. 1004-1010, June, 1987. Original article submitted April 16, 1986.

$$\left.\frac{\partial \Theta_1(x, y, z, \tau)}{\partial z}\right|_{z=0} = -\frac{q(\tau)}{\lambda} \quad (|x|,\ |y| < l,\ z = 0,\ \tau > 0); \tag{3}$$

$$\left.\frac{\partial \Theta_j(x, y, z, \tau)}{\partial z}\right|_{z=0} = 0 \quad (|x|,\ |y| > l,\ z = 0,\ \tau \geqslant 0,\ j = 2,\ 3,\ 4); \tag{4}$$

$$\left.\frac{\partial \Theta_1(x, y, z, \tau)}{\partial x}\right|_{x=0} = \left.\frac{\partial \Theta_1(x, y, z, \tau)}{\partial y}\right|_{y=0} = 0 \quad (|x|,\ |y| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{5}$$

$$\left.\frac{\partial \Theta_2(x, y, z, \tau)}{\partial y}\right|_{|y|=\infty} = 0 \quad (|x| < l,\ |y| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{6}$$

$$\left.\frac{\partial \Theta_3(x, y, z, \tau)}{\partial y}\right|_{|y|=\infty} = \left.\frac{\partial \Theta_3(x, y, z, \tau)}{\partial x}\right|_{|x|=\infty} = 0 \quad (|x|,\ |y| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{7}$$

$$\left.\frac{\partial \Theta_4(x, y, z, \tau)}{\partial x}\right|_{|x|=\infty} = 0 \quad (|x| > l,\ |y| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{8}$$

$$\Theta_1(x,\ l,\ z,\ \tau) = \Theta_2(x,\ l,\ z,\ \tau) \quad (|x| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{9}$$

$$\left.\frac{\partial \Theta_1(x, y, z, \tau)}{\partial y}\right|_{|y|=l} = \left.\frac{\partial \Theta_2(x, y, z, \tau)}{\partial y}\right|_{|y|=l} \quad (|x| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{10}$$

$$\Theta_1(l,\ y,\ z,\ \tau) = \Theta_4(l,\ y,\ z,\ \tau) \quad (|y| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{11}$$

$$\left.\frac{\partial \Theta_1(x, y, z, \tau)}{\partial x}\right|_{|x|=l} = \left.\frac{\partial \Theta_4(x, y, z, \tau)}{\partial x}\right|_{|x|=l} \quad (|y| < l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{12}$$

$$\Theta_2(l,\ y,\ z,\ \tau) = \Theta_3(l,\ y,\ z,\ \tau) \quad (|y| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{13}$$

$$\left.\frac{\partial \Theta_2(x, y, z, \tau)}{\partial x}\right|_{|x|=l} = \left.\frac{\partial \Theta_3(x, y, z, \tau)}{\partial x}\right|_{|x|=l} \quad (|y| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{14}$$

$$\Theta_3(x,\ l,\ z,\ \tau) = \Theta_4(x,\ l,\ z,\ \tau) \quad (|x| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{15}$$

$$\left.\frac{\partial \Theta_3(x, y, z, \tau)}{\partial y}\right|_{|y|=l} = \left.\frac{\partial \Theta_4(x, y, z, \tau)}{\partial y}\right|_{|y|=l} \quad (|x| > l,\ z \geqslant 0,\ \tau \geqslant 0); \tag{16}$$

$$\Theta_1(l,\ l,\ z,\ \tau) = \Theta_3(l,\ l,\ z,\ \tau) \quad (z \geqslant 0,\ \tau \geqslant 0); \tag{17}$$

$$\Theta_2(l,\ l,\ z,\ \tau) = \Theta_4(l,\ l,\ z,\ \tau) \quad (z \geqslant 0,\ \tau \geqslant 0). \tag{18}$$

The solution for the Laplace transform $\bar{\Theta}_1(0,\ 0,\ z,\ s)$ on the axis $z \geqslant 0$ $(x = y = 0)$ is written in the form (s is the Laplace transform parameter)

$$\bar{\Theta}_1(0,\ 0,\ z,\ s) = \frac{\sqrt{a}}{\lambda}\bar{q}(s)\frac{1}{\sqrt{s}}\exp\left(-\frac{z}{\sqrt{a}}\sqrt{s}\right) - \frac{\bar{q}(s)}{\pi\lambda}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{3}\left\{3\int_0^{(n+2)l} K_0\left(\frac{\sqrt{s}}{\sqrt{a}}\sqrt{p^2+z^2}\right)dp - \int_0^{(n+1)l} K_0\left(\frac{\sqrt{s}}{\sqrt{a}}\sqrt{p^2+z^2}\right)dp - 7\int_0^{(n+3)l} K_0\left(\frac{\sqrt{s}}{\sqrt{a}}\sqrt{p^2+z^2}\right)dp + 5\int_0^{(n+4)l} K_0\left(\frac{\sqrt{s}}{\sqrt{a}}\sqrt{p^2+z^2}\right)dp\right\}, \tag{19}$$

where

$$\bar{q}(s) = \int_0^{\infty} q(\tau)\exp(-s\tau)\,d\tau. \tag{20}$$

Applying the inverse Laplace transform to (19), we obtain the temperature difference $\Theta_1(0,\ 0,\ z,\ \tau)$ for $x = y = 0$, $z \geqslant 0$, $\tau > 0$ in the form:

$$\Theta_1(0,\ 0,\ z,\ \tau) = \frac{1}{b\sqrt{\pi}}\int_0^{\tau}\exp\left[-\frac{z^2}{4a(\tau-\xi)}\right]\frac{q(\xi)\,d\xi}{\sqrt{\tau-\xi}} -$$

$$-\frac{1}{b\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{6}\left\{\int_0^{\tau}\exp\left(-\frac{z^2}{4a(\tau-\xi)}\right)\times\right.$$

$$\times\left[\operatorname{erfc}\frac{(n+1)l}{2\sqrt{a(\tau-\xi)}}-3\operatorname{erfc}\frac{(n+2)l}{2\sqrt{a(\tau-\xi)}}+\right.$$

$$\left.\left.+7\operatorname{erfc}\frac{(n+3)l}{2\sqrt{a(\tau-\xi)}}-5\operatorname{erfc}\frac{(n+4)l}{2\sqrt{a(\tau-\xi)}}\right]\frac{q(\xi)\,d\xi}{\sqrt{\tau-\xi}}\right\}. \tag{21}$$

Equation (21) evaluated at $z = 0$ gives an expression for the temperature difference at the point $x = y = z = 0$ (the origin of the Cartesian coordinate system) when a heat flux density $q(\xi) = q(\tau)$ acts over a finite square region on the surface of the body:

$$\Theta_1(0,\ 0,\ 0,\ \tau)=\frac{1}{b\sqrt{\pi}}\int_0^{\tau}\frac{q(\xi)}{\sqrt{\tau-\xi}}\,d\xi-\frac{1}{b\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(n+1)}{6}\times$$

$$\times(n+2)(n+3)\left\{\int_0^{\tau}\frac{q(\xi)}{\sqrt{\tau-\xi}}\left[\operatorname{erfc}\frac{(n+1)l}{2\sqrt{a(\tau-\xi)}}-\right.\right.$$

$$\left.\left.-3\operatorname{erfc}\frac{(n+2)l}{2\sqrt{a(\tau-\xi)}}+7\operatorname{erfc}\frac{(n+3)l}{2\sqrt{a(\tau-\xi)}}-5\operatorname{erfc}\frac{(n+4)l}{2\sqrt{a(\tau-\xi)}}\right]d\xi\right\} \tag{22}$$

Specifying the time dependence of the heat flux density $q(\tau)$ in the square region (of side $2l$) on the surface of the semiinfinite body, and integrating (21) and (22), we obtain a series of particular solutions for $\Theta_1(0,\ 0,\ z,\ \tau)$ and $\Theta_1(0,\ 0,\ 0,\ \tau)$. We assume that the heat flux density in the square region on the surface of the body is constant in time, i.e., $q(\tau) = q_0 = \text{const}$. Then it is not difficult to obtain from (22) an expression for the temperature difference on the surface at a point $x = y = z = 0$ in the form:

$$\Theta_1(0,\ 0,\ 0,\ \tau)=\frac{2q_0\sqrt{\tau}}{b\sqrt{\pi}}-\frac{q_0\sqrt{\tau}}{b\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{3}\times$$

$$\times\left\{\operatorname{erfc}\frac{(n+1)l}{2\sqrt{a\tau}}-3\operatorname{erfc}\frac{(n+2)l}{2\sqrt{a\tau}}+7\operatorname{erfc}\frac{(n+3)l}{2\sqrt{a\tau}}-\right.$$

$$\left.-5\operatorname{erfc}\frac{(n+4)l}{2\sqrt{a\tau}}\right\}+\frac{q_0l}{\pi\lambda}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{6}\times$$

$$\times\left\{(n+1)E_1\left[\frac{(n+1)^2l^2}{4a\tau}\right]-3(n+2)E_1\left[\frac{(n+2)^2l^2}{4a\tau}\right]+\right.$$

$$\left.+7(n+3)E_1\left[\frac{(n+3)^2l^2}{4a\tau}\right]-5(n+4)E_1\left[\frac{(n+4)^2l^2}{4a\tau}\right]\right\}, \tag{23}$$

where $E_1(Z) = -\mathrm{Ei}(-Z)$ is the exponential integral [1]. If we introduce the dimensionless quantities

$$\Theta_1^*(0,\ 0,\ 0,\ \mathrm{Fo})=\frac{\Theta_1(0,\ 0,\ 0,\ \mathrm{Fo})}{T_0},\quad \mathrm{Ki}=\frac{q_0l}{\lambda T_0},\quad \mathrm{Fo}=\frac{a\tau}{l^2},$$

then we can write the solution (23) in the form

$$\frac{\Theta_1^*(0,\ 0,\ 0,\ \mathrm{Fo})}{\mathrm{Ki}}=\frac{2\sqrt{\mathrm{Fo}}}{\sqrt{\pi}}-\frac{\sqrt{\mathrm{Fo}}}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{3}\times$$

$$\times\left\{\operatorname{erfc}\frac{(n+1)}{2\sqrt{\mathrm{Fo}}}-3\operatorname{erfc}\frac{(n+2)}{2\sqrt{\mathrm{Fo}}}+7\operatorname{erfc}\frac{(n+3)}{2\sqrt{\mathrm{Fo}}}-\right.$$

$$\left.-5\operatorname{erfc}\frac{(n+4)}{2\sqrt{\mathrm{Fo}}}\right\}+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)(n+3)}{6}\times$$

$$\times \left\{ (n+1)\, E_1 \left[\frac{(n+1)^2}{4\,\text{Fo}} \right] - 3\,(n+2)\, E_1 \left[\frac{(n+2)^2}{4\,\text{Fo}} \right] + \right.$$
$$\left. + 7\,(n+3)\, E_1 \left[\frac{(n+3)^2}{4\,\text{Fo}} \right] - 5\,(n+4)\, E_1 \left[\frac{(n+4)^2}{4\,\text{Fo}} \right] \right\}. \tag{24}$$

In the limit $l \to \infty$, (23) gives an expression for the one-dimensional temperature field $\Theta_{\text{odf}}(0, \tau) = T_{\text{odf}}(0, \tau) - T_0 = \lim_{l \to \infty} \Theta_1(0, 0, 0, \tau) = 2q_0 \sqrt{a\tau}/(\lambda \sqrt{\pi}) = 2q_0 \sqrt{\tau}/(b\sqrt{\pi})$, where $b = \lambda/\sqrt{a}$ is the thermal activity of the body. Hence the function

$$\delta(\text{Fo}) = \frac{1}{2} \sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)}{3} \left\{ \operatorname{erfc} \frac{(n+1)}{2\sqrt{\text{Fo}}} - \right.$$
$$\left. - 3 \operatorname{erfc} \frac{(n+2)}{2\sqrt{\text{Fo}}} + 7 \operatorname{erfc} \frac{(n+3)}{2\sqrt{\text{Fo}}} - 5 \operatorname{erfc} \frac{(n+4)}{2\sqrt{\text{Fo}}} \right\} -$$
$$- \frac{1}{2\sqrt{\pi\,\text{Fo}}} \sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)}{6} \left\{ (n+1)\, E_1 \left[\frac{(n+1)^2}{4\,\text{Fo}} \right] - \right.$$
$$\left. - 3\,(n+2)\, E_1 \left[\frac{(n+2)^2}{4\,\text{Fo}} \right] + 7\,(n+3)\, E_1 \left[\frac{(n+3)^2}{4\,\text{Fo}} \right] - 5(n+4) E_1 \left[\frac{(n+4)^2}{4\,\text{Fo}} \right] \right\} \tag{25}$$

will give the relative deviation between the temperature $\Theta_1(0, 0, 0, \text{Fo})$ in the three-dimensional case and the corresponding temperature in the one-dimensional case, for the special case of a constant specific heat flux. Graphs of the function (25) are shown in Fig. 2.

The results (23) and (24) can be used to develop a nondestructive method of determining the thermal and physical characteristics of materials (without destroying the integrity of the sample) if a square heat source with constant power and small heat capacity is placed between two identical bodies in contact with the source, and the temperature excess is measured at the center of the square heated from the instant the source is turned on.

When $\text{Fe} \leqslant 1$ the relative deviation $\delta(\text{Fo}) \leqslant 0.003122$. Then the thermal activity b can be determined from the formula

$$b = \frac{2q_0 \sqrt{\tau}}{\sqrt{\pi}\, \Theta_1(0, 0, 0, \tau)}. \tag{26}$$

We note that in the calculation of q_0, the total heat power generated by the heater will be divided equally between the two bodies when the constant power source is placed symmetrically between the two identical bodies.

The thermal diffusivity can be determined from (24) by calculating the ratios of the temperatures at different instants of time $\tau > 0$ ($\text{Fo} > 0$). The traditional method of determining a from the measured time dependence of the temperature at a single point of the sample uses the experimental values of the ratio $N' = \Theta_1(0, 0, 0, p\tau_1)/\Theta_1(0, 0, 0, \tau_1) = \Theta_1^*(0, 0, 0, p\text{Fo}_1)/\Theta_1^*(0, 0, 0, \text{Fo}_1) = f(p, \text{Fo}_1)$, where $p = \tau_2/\tau_1 > 1$, where it is assumed that multiple measurements of the temperature excess (23) are available. The subsequent procedure then reduces to an analytical method of determining the argument Fo_1 from the known values of N' and p.

The thermal diffusivity is calculated from the formula

$$a = \frac{l^2}{\tau_1} \text{Fo}_1. \tag{27}$$

The thermal conductivity and heat capacity per unit volume are determined from

$$\lambda = b\sqrt{a};\; c\gamma = \frac{\lambda}{a} = \frac{b}{\sqrt{a}}. \tag{28}$$

A graph of $\Theta_1^*(0, 0, 0, \text{Fo})/\text{Ki} = \Theta_1^*/\text{Ki} = f(\text{Fo})$ is shown in Fig. 3. For practical calculations Table 1 gives the values of this function for Fo in steps of 0.01, such that an accuracy of (24) up to sixth figures beyond the decimal place is obtained.

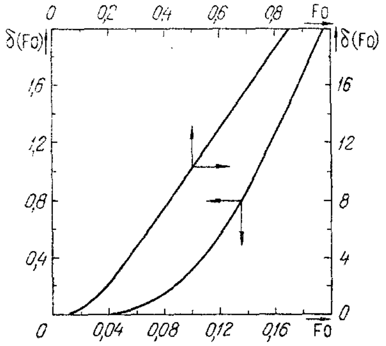

Fig. 2

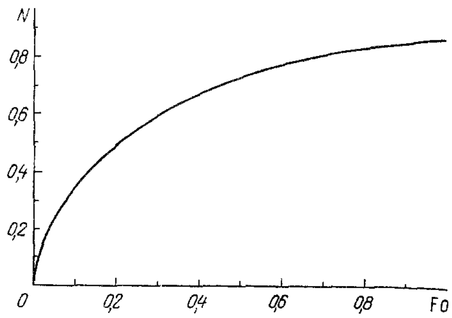

Fig. 3

Fig. 2. Dependence of the relative percent deviation (error) δ(Fo) calculated from (25) on the number Fo.

Fig. 3. Dependence of $N = \Theta_1^*(0, 0, 0, \mathrm{Fo})/\mathrm{Ki} = f(\mathrm{Fo})$ (from (24)) on Fo.

TABLE 1. Dependence of the Function $\Theta_1^*(0, 0, 0, \mathrm{Fo})/\mathrm{Ki} = f(\mathrm{Fo})$ (from (24)) on Fo

Fo	0	1	2	3	4
0,00	0	0,035682	0,050462	0,061803	0,071364
0,0	0	0,112837	0,159576	0,195440	0,225670
0,1	0,355710	0,372620	0,388644	0,403877	0,418400
0,2	0,493723	0,504685	0,515276	0,525523	0,535433
0,3	0,588949	0,597022	0,604844	0,612495	0,619929
0,4	0,660725	0,666966	0,673045	0,678967	0,684796
0,5	0,716782	0,721708	0,726530	0,731182	0,735767
0,6	0,761068	0,764825	0,768611	0,772369	0,775852
0,7	0,795507	0,798416	0,801234	0,804047	0,806827
0,8	0,821327	0,823465	0,825545	0,827429	0,829375
0,9	0,839346	0,840783	0,841942	0,843196	0,844363
1,0	0,849794	0,850544	0,851117	0,851559	0,851888

Fo	5	6	7	8	9
0,00	0,079788	0,087403	0,094406	0,100925	0,107047
0,0	0,252283	0,276303	0,298325	0,318734	0,337798
0,1	0,432279	0,445563	0,458313	0,470565	0,482359
0,2	0,545040	0,554348	0,563375	0,572159	0,580670
0,3	0,627165	0,634216	0,641090	0,647808	0,654335
0,4	0,690423	0,695974	0,700553	0,706629	0,711800
0,5	0,740254	0,744608	0,748838	0,753015	0,757094
0,6	0,779367	0,782802	0,786091	0,789248	0,792417
0,7	0,809461	0,811985	0,814391	0,816757	0,819127
0,8	0,831383	0,833078	0,834612	0,836208	0,837775
0,9	0,845511	0,846485	0,847420	0,848192	0,849063
1,0	0,852280	0,852591	0,853069	0,852966	0,853047

This method of determining the thermal and physical characteristics assumes a thin square heater of constant power. From the point of view of the difficulty of making a wire heater in various geometrical shapes (such as a square, circle, ring, etc.) a square is preferable in our opinion.

NOTATION

$\Theta_1(x, y, x, \tau)$, temperature field in a semiinfinite body of the region $|x| < l$, $|y| < l$, $z \geqslant 0$, $\tau > 0$; $\Theta_2(x, y, z, \tau)$, temperature field in the region $|x| < l$, $|y| > l$, $z \geqslant 0$, $\tau > 0$; $\Theta_3(x, y, z, \tau)$, temperature field in the region $|x| > l$, $|y| > l$, $z \geqslant 0$, $\tau > 0$; $\Theta_4(x, y, z, \tau)$, temperature field in the region $|x| > l$, $|y| < l$, $z \geqslant 0$, $\tau > 0$; $q(\tau)$, heat flux density in a prescribed square region on the surface of the body; a, $c\gamma$, λ, b, thermal diffusivity, heat capacity per unit volume, thermal conductivity, and thermal activity of the body, respectively; τ, time; ∇^2, Laplacian in Certesian coordinates x, y, z; s, Laplace transform parameter; $K_0(X)$, zero-order modified Bessel function of the second kind (the MacDonald function); erfc(X), complement of the error function; $E_i(X) = -Ei(-X)$, exponential integral; $\mathrm{Ki} = q_0 l/(\lambda T_0)$, Kirpichev number; $\mathrm{Fo} = a\tau/l^2$, Fourier number; $\Theta_1^*(0, 0, 0, \mathrm{Fo}) = \Theta_1(0, 0, 0, \mathrm{Fo})/T_0$, dimensionless relative temperature; $N = \Theta_1^*(0, 0, 0, \mathrm{Fo})/\mathrm{Ki}$, discrete value of (24); see text; δ(Fo), relative deviation of the temperature excess $\Theta_1^*(0, 0, 0, Fo)$ for the three-dimensional case from the corresponding quantity in the one-dimensional case; q_0, constant heat flux density inside the prescribed square region on the surface of the body; T_0, initial temperature of the body; $2l$, length of a side of the square heater; τ_1, time corresponding to Fo_1.

LITERATURE CITED

1. V. P. Kozlov, Two-Dimensional Axisymmetric Unsteady Problems of Heat Conduction [in Russian], Minsk (1986).
2. G. M. Serykh, B. P. Kolesnikov, and V. G. Sysoev, Prom. Teplotekh., 3, No. 1, 85-91 (1981).
3. V. V. Vlasov, Yu. S. Shatalov, E. N. Zotov, et al., Izmerit. Tekh., No. 6, 42-45 (1980).

SANDWICH PLATE UNDER THERMAL IMPACT

V. V. Kharitonov, T. A. Starovoitova, and É. I. Starovoitov

UDC 536.24:534.1

An expression is obtained for the temperature field and the fluctuations excited by a thermal impact are investigated.

The extensive application of laminated structure elements in industry arouses interest in determining the temperature fields therein and in describing their dynamic behavior under thermal force action. The vibrations of a circular single-plate excited by a thermal impact are considered in the monograph [1]. Similar investigations are performed in this paper for sandwich circular plates of nonsymmetrical thickness, assembled from materials with different thermophysical and mechanical properties.

Let us consider an unlimited sandwich plate of thickness $h = \sum_{k=1}^{3} h_k$ (k = 1, 2, 3; h_1, h_2 are the thicknesses of the outer layers and $h_3 = 2c$ is the filler thickness), on whose outer surface of the heat shielding layer 1 ($z = c + h_1$) a thermal flux of density q_t acts in a direction normal to the surface. The outer plane of the carrying layer 2 ($z = -c - h_2$) is assumed heat-insulated. A cylindrical r, φ, z coordinate system is coupled to the filler middle surface, and the z axis is directed toward the layer 1. Under the mentioned heat-transfer conditions the temperature field in the k-th layer of the plate $\theta_k(z, t) = T - T_0$ (T_0 is the initial temperature) satisfies the heat conduction equation

$$\theta_{k,zz} = \dot{\theta}_k / a_{kt}, \tag{1}$$

under the initial (t = 0, t is the time)

$$\theta_k(z, 0) = 0 \tag{2}$$

and boundary

$$\left.\begin{aligned} &\lambda_1\theta_{1,z} = -q_t \ (z = c + h_1),\ \theta_1 = \theta_3,\ \lambda_1\theta_{1,z} = \lambda_3\theta_{3,z} \ (z = c), \\ &\theta_2 = \theta_3,\ \lambda_2\theta_{2,z} = \lambda_3\theta_{3,z} \ (z = -c),\ \theta_{2,z} = 0 \ (z = -c - h_2) \end{aligned}\right\} \tag{3}$$

conditions. Here $a_{kt} = \lambda_k/(c_{kt}\rho_k)$ is the thermal diffucivity of the k-th layer, the comma in the subscript denotes the operation of differentiation with respect to the subsequent coordinates.

The solution of the boundary-value problem (1) under the initial (2) and boundary (3) condition is executed by an operational method based on the Laplace transform [2].

Analysis of the analytical expressions obtained for the temperature fields in each of the layers and comparing them with known solutions (for $h_2 = 0$ the field for a bilayer plate presented in [2] follows, while for $h_1 \ll h_3$ we obtain the temperature field of a thin coat-

Belorussian Institute of Railroad Transport Engineers, Gomel'. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 6, pp. 1010-1017, June, 1987. Original article submitted April 16, 1986.

 0022-0841/87/5206-0734 $12.50